Straight Lines and Pair of Straight Lines

Question1

The system of equations $4x + 6y = 5$ and $8x + 12y = 10$ has

KCET 2025

Options:

A. No solution

B. Infinitely many solutions

C. A unique solution

D. Only two solutions

Answer: B

Solution:

Notice that the second equation is just twice the first:

First equation:

$4x + 6y = 5$

Multiply both sides by 2:

$8x + 12y = 10$

Since both equations represent the same line, there are infinitely many solutions.

Answer: Option B.

Question2

A line passes through $(-1, -3)$ and perpendicular to $x + 6y = 5$. Its x intercept is

KCET 2025

Options:

A. $\frac{1}{2}$

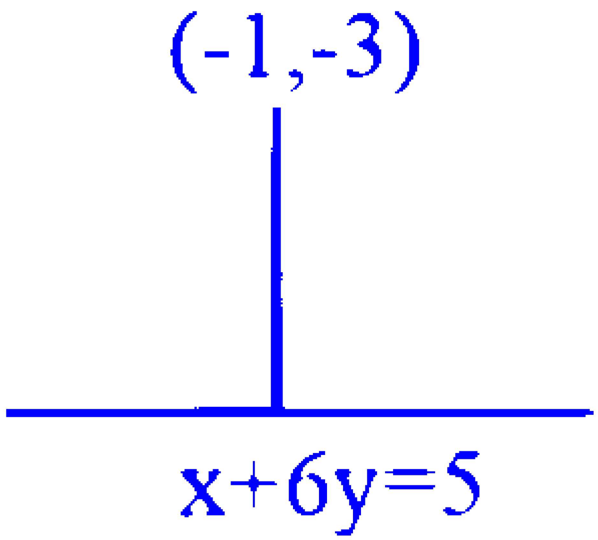
B. $-\frac{1}{2}$

C. -2

D. 2

Answer: B

Solution:



$y + 3 = 6(x + 1)$
x intercept $y = 0$
$x = \dfrac{-1}{2}$

Question3

The angle between the line $x + y = 3$ and the line joining the points $(1, 1)$ and $(-3, 4)$ is

KCET 2024

Options:

A. $\tan^{-1}(7)$

B. $\tan^{-1}\left(-\frac{1}{7}\right)$

C. $\tan^{-1}\left(\frac{1}{7}\right)$

D. $\tan^{-1}\left(\frac{2}{7}\right)$

Answer: C

Solution:

$\because y = -x + 3; m_1 = -1 = \tan\theta_1$

and $m_2 = \frac{y-1}{x-1} = \frac{4-1}{-3-1}$

$\frac{y-1}{x-1} = \frac{3}{-4}; m_2 = \frac{3}{-4} = \tan\theta_2$

Angle between two lines are

$$= \tan|\theta_2 - \theta_1| = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_1 \cdot \tan\theta_2} = \frac{m_2 - m_1}{1 + m_1 m_2}$$
$$= \frac{\frac{-3}{4} + 1}{1 + \frac{3}{4}} = \frac{1}{4} \times \frac{4}{7}$$

Hence, $|\theta_2 - \theta_1| = \tan^{-1}\left(\frac{1}{7}\right)$

Question4

A line passes through $(2, 2)$ and is perpendicular to the line $3x + y = 3$. Its y-intercept is

KCET 2023

Options:

A. $\frac{2}{3}$

B. 1

C. $\frac{4}{3}$

D. $\frac{1}{3}$

Answer: C

Solution:

Equation of the line passing through $(2,2)$ and perpendicular to $3x+y=3$

$$(x-2)-3(y-2)=0$$
$$\Rightarrow \quad x-3y=-4$$

$\therefore y$ intercept of this line is $\frac{4}{3}$.

Question5

If the straight line $2x-3y+17=0$ is perpendicular to the line passing through the points $(7,17)$ and $(15,\beta)$, then β equals

KCET 2022

Options:

A. -5

B. 5

C. 29

D. -29

Answer: B

Solution:

Equation of line passing through $(7, 17)$ and $(15, \beta)$ is

$$(y-17)=\left(\frac{\beta-17}{15-7}\right)(x-7)$$
$$\Rightarrow \quad y-17=\frac{(\beta-17)}{8}(x-7)$$
$$\Rightarrow \quad y=\left(\frac{\beta-17}{8}\right)x-\frac{7(\beta-17)}{8}+17$$

On comparing to $y=mx+c$, we get Slope, $m_1=\frac{\beta-17}{8}$

It is given that above line is perpendicular to

$2x-3y+17=0$

$\Rightarrow \quad y=\frac{2}{3}x+\frac{17}{3}$

$\therefore$ Slope, $m_2=\frac{2}{3}$

Condition for perpendicular lines, $m_1 \cdot m_2=-1$

$$\left(\frac{\beta-17}{8}\right)\times\frac{2}{3}=-1$$
$$\Rightarrow \quad \beta-17=-12 \Rightarrow \beta=5$$

Question6

If the vertices of a triangle are $(-2, 6)$, $(3, -6)$ and $(1, 5)$, then the area of the triangle is

KCET 2022

Options:

A. 40 sq. units

B. 15.5 sq. units

C. 30 sq. units

D. 35 sq. units

Answer: B

Solution:

Given vertices of triangle are $(-2,6),(3,-6)$ and $(1,5)$.

Area of triangle $=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$=\frac{1}{2}\begin{vmatrix} -2 & 6 & 1 \\ 3 & -6 & 1 \\ 1 & 5 & 1 \end{vmatrix}=\frac{1}{2}\begin{vmatrix} -5 & 12 & 0 \\ 2 & -11 & 0 \\ 1 & 5 & 1 \end{vmatrix}$$

$[R_1 \leftrightarrow R_1 - R_2, R_2 \leftrightarrow R_2 - R_3]$

$=\frac{1}{2}[1(55-24)]=\frac{31}{2}=15.5$ sq. units

Question7

The equation of straight line which passes through the point $\left(a\cos^3\theta, a\sin^3\theta\right)$ and perpendicular to $x\sec\theta + y\,\text{cosec}\,\theta = a$ is

KCET 2021

Options:

A. $\frac{x}{a}+\frac{y}{a}=a\cos\theta$

B. $x\cos\theta - y\sin\theta = a\cos 2\theta$

C. $x\cos\theta + y\sin\theta = a\cos 2\theta$

D. $x\cos\theta - y\sin\theta = a\cos\theta$

Answer: B

Solution:

Given,

Point $(x_1, y_1) = (a\cos^3\theta, a\sin^3\theta)$

Other line $\Rightarrow x\sec\theta + y\,\text{cosec}\,\theta = a$

$$y = -\frac{x\sec\theta}{\text{cosec}\,\theta} + \frac{a}{\text{cosec}\,\theta}$$

$$\Rightarrow \quad y = \left(-\frac{\sin\theta}{\cos\theta}\right)x + a\sin\theta$$

$$\Rightarrow \quad y = m_1 x + c$$

$$\therefore \quad m_1 = -\frac{\sin\theta}{\cos\theta}$$

Equation of required line

$\Rightarrow (y - y_1) = m(x - x_1)$

where $m = \frac{\cos\theta}{\sin\theta}\left[\because m_1 \cdot m = -1, m = -\frac{1}{m_1} = -\frac{1}{\left(-\frac{\sin\theta}{\cos\theta}\right)} = \frac{\cos\theta}{\sin\theta}\right]$

$$(y - a\sin^3\theta) = \frac{\cos\theta}{\sin\theta}(x - a\cos^3\theta)$$

$$\Rightarrow \quad y\sin\theta - a\sin^4\theta = x\cos\theta - a\cos^4\theta$$

$$\Rightarrow \quad x\cos\theta - y\sin\theta = a(\cos^4\theta - \sin^4\theta)$$

$$= a(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$$

$$= a \cdot (1) \cdot (\cos 2\theta) = a\cos 2\theta$$

$$\Rightarrow \quad x\cos\theta - y\sin\theta = a\cos 2\theta$$

Question8

The two lines $lx + my = n$ and $l'x + m'y = n'$ are perpendicular if

KCET 2020

Options:

A. $ll' + mm' = 0$

B. $lm' + ml'$

C. $lm + l'm' = 0$

D. $lm' + ml' = 0$

Answer: A

Solution:

Given lines

$lx + my = n$ and $lx + m'y = n'$

Slope of line $lx + my - n = 0$ is $-l/m$ and slope of line $lx + m'y - n' = 0$ is $-l'/m'$ lines are perpendicular

$$\therefore \quad \left(\frac{-l}{m}\right)\left(\frac{-l'}{m'}\right) = -1$$
$$ll' + mm' = 0$$

Question9

A line cuts off equal intercepts on the co-ordinate axes. The angle made by this line with the positive direction of X-axis is

KCET 2019

Options:

A. 120°

B. 45°

C. 135°

D. 90°

Answer: C

Solution:

Equation of line which cuts off equal intercepts on co-ordinate axes,

$$x + y = a \Rightarrow y = -x + a$$
$$\text{Here, } \tan\theta = -1 = \tan 135^\circ \Rightarrow \theta = 135^\circ$$

Question10

The equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through $(-2, 3)$ is

KCET 2018

Options:

A. $3x - 4y + 18 = 0$

B. $3x + 4y + 18 = 0$

C.

$3x - 4y - 18 = 0$

D.

$3x + 4y - 18 = 0$

Answer: A

Solution:

To find the equation of a line parallel to the given line $3x - 4y + 2 = 0$ that passes through the point $(-2, 3)$, follow these steps:

Identify the form of the parallel line:

A line parallel to $3x - 4y + 2 = 0$ will have the same coefficients for x and y. Thus, the equation of the line we are looking for can be written as:

$3x - 4y = \lambda$

Determine λ using the given point:

Since the line passes through the point $(-2, 3)$, substitute these coordinates into the equation:

$3(-2) - 4(3) = \lambda$

Calculate λ:

$-6 - 12 = \lambda \implies \lambda = -18$

Write the equation of the line:

Substitute $\lambda = -18$ back into the equation of the line:

$3x - 4y = -18$

Express the equation in standard form:

Rewrite the equation as:

$3x - 4y + 18 = 0$

Therefore, the equation of the line parallel to the given line and passing through the point $(-2, 3)$ is $3x - 4y + 18 = 0$.

Question11

Equation of line passing through the point $(1, 2)$ and perpendicular to the line $y = 3x - 1$

KCET 2017

Options:

A. $x + 3y = 0$

B. $x + 3y - 7 = 0$

C. $x + 3y + 7 = 0$

D. $x - 3y = 0$

Answer: B

Solution:

Given the line equation:

$y = 3x - 1$

The slope of this line is 3.

To find the equation of a line perpendicular to this, we need the negative reciprocal of the given line's slope. Thus, the slope of the perpendicular line is:

$-\frac{1}{3}$

The equation of a line with this slope that passes through the point $(1, 2)$ can be set up using the point-slope form:

$y - 2 = -\frac{1}{3}(x - 1)$

Expanding this equation, we get:

$3(y - 2) = -(x - 1)$

Which simplifies to:

$3y - 6 = -x + 1$

Rearranging terms, we arrive at:

$x + 3y - 7 = 0$

This is the equation of the line that passes through the point $(1, 2)$ and is perpendicular to $y = 3x - 1$.